( An Institute of Advanced Mathematics )

Roll No.

## MATHEMATI CS <br> SET - A

## Time allowed : 3 hr

Maximum Marks: 100

## General Instructions:

(i) All Question are compulsory.
(ii) The question paper consists of 29 questions divided into three sections $\mathrm{A}, \mathrm{B}$, and C . Section A comprises of 10 questions of one mark each. Section B comprises of 12 questions of four marks each and Section C comprises of $\mathbf{7}$ questions of six marks each.
(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question..
(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted

## SECTION A

1. Find the sum: $0.15,0.015,0.0015, \ldots . .20$ terms
2. Find the Value of $\operatorname{Cos}\left(-1710^{\circ}\right)$
3. Find the argument of the complex number $\frac{1+i}{1-i}$
4. Solve $\tan 2 \mathrm{x}=-\cot \left(x+\frac{\pi}{3}\right)$
5. In a race there are five teams $A, B, C, D$ and $E$. What is the probability that $A, B$ and $C$ are first three to finish (in any order) (Assume that all finishing orders are equally likely)
6. Solve: $\sqrt{3} x^{2}-\sqrt{2} x+3 \sqrt{3}=0$
7. Find the equation of a line perpendicular to the line $x-2 y+3=0$ and passing through the point $(1,-2)$.
8. Find $n$ if $/ n-1 P_{3}: n R_{4}=1.9$.
9. Write the general term in the expansion of $\left(x^{2}-y x\right)^{12}, x \neq 0$.]
10. Find $n$ if ${ }^{n-1} P_{3}:{ }^{n} P_{4}=1: 9$.

## SECTION B

11. Find real $\theta$ such that $\frac{3+2 i \sin \theta}{1-2 i \sin \theta}$ is purely real.
12. Prove by the principle of mathematical induction: $(1+x)^{n} \geq(1+n x)$, for all natural number $n$, where $x>-1$.
13. How many numbers lying between 100 and 1000 can be formed with the digits $0,1,2,3,4,5$ if the repetition of the digits is not allowed.
14. How many numbers greater than 100000 can be formed by using the digits $1,2,0,2,4,2,4$

15 Solve $\sin 2 x-\sin 4 x+\sin 6 x=0$
16. Find $\sin \frac{x}{2}, \cos \frac{x}{2}$ and $\tan \frac{x}{2}$ : if $\cos =-\frac{1}{3}$, and x lies in quadrant III.

OR
$2 \tan A=3 \tan B$ Prove that $\tan (A-B)=\frac{\sin 2 B}{5-\cos 2 B}$

17 Find the direction in which a straight line must be drawn through the point $(-1,2)$ so that the point of intersection with the line $x+y=4$ may be at a distance of 3 units from this point.

## OR

Find the equation of the circle passing through the points $(4,1)$ and $(6,5)$ and whose centre is on the line $4 x+y=16$
18 A man running a racecourse notes that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m . Find the equation of the posts traced by a man.

## OR

Find the equation of the ellipse, the distance between whose foci is 8 units and the distance between the directories is 18 units.

19 Show that two lines $a_{1} x+b_{1} y+c_{1}=0$ and $a_{2} x+b_{2} y+c_{2}=0$, where $b_{1}, b_{2} \neq 0$ are:
(i) Parallel if $\frac{a_{1}}{b_{1}}=\frac{a_{2}}{b_{2}}$ and (ii) Perpendicular if $\mathrm{a}_{1} \mathrm{a}_{2}+\mathrm{b}_{1} \mathrm{~b}_{2}=0$.

20 Define a relation $R$ defined on $A=\{0,1,2,4,5\}$ from $A$ to $A$ by $R=\{(x, y): y=x+5$ where $x, y \in A\}$.
Write $R$ in Roaster form and hence find its Domain, Co domain and Range.
21 Find the ratio in which YZ-plane divides the line segment formed by joining the points $(-2,4,7) \&(-3,5,8)$
22 If $A, B, C$ are three events associated with a random experiment, prove that

$$
P(A \cup B \cup C)=P(A)+P(B)+P(C)-P(A \cap B)-P(A \cap C)-P(B \cap C)+P(A \cap B \cap C)
$$

If 4 -digit numbers greater than 5,000 are randomly formed from the digits $0,1,3,5$ and 7 , what is the probability of forming a number divisible by 5 when, (i) the repetition of digits is not/allowed, (ii) the digits are repeated?

## SECTION - C

23. In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking
(1) neither tea nor coffee.
(ii) Only Tea
(iii) Only Coffee
24. Solve the equation graphically and name the yertices of the feasible region alongwith their coordinates $x+2 y \leq 6, \quad 2 x+y \leq 6, \quad x \geq 1, y \geq-1$.
25. Find the sum of the following series up to n terms: $\frac{1^{3}}{1}+\frac{1^{3}+2^{3}}{1+3}+\frac{1^{3}+2^{3}+3^{3}}{1+3+5} \ldots$.

If $A$ and G be A. M. and G.M., respectively between two positive numbers, prove that the numbers are $7 \mathrm{~A} \pm \sqrt{(A+G)(A-G)}$
26. If the coefficients of $a^{r-1}$, $a^{r}$ and $a^{r+1}$ in the expansions of $(1+a)^{n}$ are in arithmetic progression, prove that $n^{2}-n(4 r+1)+4 r^{2}-2=0$

## OR

If the coefficient of $(r-5)^{\text {th }}$ and $(2 r-1)^{\text {th }}$ terms of the expansion of $(1+x)^{34}$ are equal, find $r$.
27. (a) For the function $\mathrm{f}(\mathrm{x})=\frac{x^{100}}{100}+\frac{x^{99}}{99}+\ldots .+\frac{x^{2}}{2}+x+1$. Prove that $\mathrm{f}^{\prime}(1)=100 \mathrm{f}^{\prime}(0)$.
(b) Find $\lim _{x \rightarrow 1}\left(\frac{1}{x^{2}+x-2}-\frac{x}{x^{3}-1}\right)$
28. The mean and standard deviation of 100 observations were calculated as 20 and 3, respectively. Later on it was found that three observations were incorrect, which was recorded as 21,21 and 18. Find the Mean and Standard Deviation if the incorrect observation are omitted.
29. Prove that: $\frac{(\sin 7 x+\sin 5 x)+(\sin 9 x+\sin 3 x)}{(\cos 7 x+\cos 5 x)+(\cos 9 x+\cos 3 x)}=\tan 6 x$

